RCNP-Th01006

Casimir scaling in a dual superconducting scenario of confinement

Y. Koma[1,2*], E. -M. Ilgenfritz[1,3], H. Toki[1], and T. Suzuki[2]

[1] *Research Center for Nuclear Physics (RCNP), Osaka University, Mihogaoka 10-1, Ibaraki, Osaka 567-0047, Japan*

[2] *Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan*

[3] *Institut für Theoretische Physik, Universität Tübingen, D-72076 Tübingen, Germany*

(February 1, 2008)

Abstract

The string tensions of flux tubes associated with static charges in various SU(3) representations are studied within the dual Ginzburg-Landau (DGL) theory. The ratios of the string tensions between higher and fundamental representations, $d_D \equiv \sigma_D/\sigma_F$, are found to depend only on the Ginzburg-Landau (GL) parameter, $\kappa = m_\chi/m_B$, the mass ratio between monopoles m_χ and dual gauge bosons m_B. In the case of the Bogomol'nyi limit ($\kappa = 1$), analytical values of d_D are easily obtained by adopting the manifestly Weyl invariant formulation of the DGL theory, which are provided simply by the number of color-electric Dirac strings inside the flux tube. A numerical investigation of the ratio for various GL-parameter cases is also performed, which suggests that the Casimir scaling is obtained in the type-II parameter range within the interval $\kappa = 5 \sim 9$ for various ratios d_D.

Key Word: Casimir scaling, dual Ginzburg-Landau theory, flux tube, string tension, Weyl symmetry

PACS number(s): 12.38.Aw, 12.38.Lg

*Email address: koma@rcnp.osaka-u.ac.jp

I. INTRODUCTION

The observation of Casimir scaling is an important argument in any discussion of the virtues of different QCD vacuum models as far as the respective confinement mechanism is concerned. Taken literally, the Casimir scaling suggests that the potential at intermediate distances between static charges in different representation are proportional to the eigenvalues $C^{(2)}(D)$ of the quadratic Casimir operator T^aT^a in the respective D dimensional representation, such that $F_{D_1}(r)/F_{D_2}(r) = C^{(2)}(D_1)/C^{(2)}(D_2)$ at all distances. This property is obvious only for the one-gluon exchange component of the static force. Although there is no asymptotically linearly rising potential for the higher representations, at intermediate distances a string tension can be defined which enters $F_D(r)$ as a constant part. The first lattice indications for the Casimir scaling appeared in the eighties [1,2]. At that time this observation was a challenge for the bag model [3]. For example, the ratio of string tensions of adjoint to fundamental charges in SU(3) gauge theory, respectively, would be $\sigma_{\rm adj}/\sigma_{\rm fund} = 9/4 = 2.25$.

Recently, as a contribution to the discussion of competing confinement mechanisms, Ref. [4] appeared where the string tensions of the fundamental and higher representations have been calculated in pure SU(3) lattice gauge theory, and the ratio was obtained nearly equal to 2, already rather close to 9/4. In Ref. [5] Bali has studied the ratios of entire interaction potentials (including Coulomb and constant terms in addition to the linear term) also for quenched SU(3) gauge theory, and in the case of adjoint and fundamental charges the ratio turned out to be very close to 9/4. All detailed (microscopic) mechanisms of confinement find it hard to explain the Casimir scaling, while it appears more natural from the point of view of the semi-phenomenological Stochastic Vacuum Model [6]. If the confinement mechanism is described by center vortices, approximate Casimir scaling for the potential can be achieved by introducing a finite thickness of the vortex, as demonstrated for the case of SU(2) lattice gauge theory [7], although the original center vortex picture gives a strictly vanishing potential for pairs of charges which transform trivially under Z_N center of the gauge group.

For the dual superconductor scenario of confinement [8,9], practically realized in the form of the dual Ginzburg-Landau (DGL) theory [10], one tends to believe that it would be difficult to accommodate Casimir scaling in this framework. Indeed, in the Abelian projection scheme [11] for SU(3) gluodynamics the long range forces are transmitted only by "diagonal gluons" which couple to charges only via T^3 and T^8. This makes it hard to understand why the Casimir scaling should hold in Abelian projected gluodynamics. For example for the ratio between adjoint and fundamental forces one would naively expect the Abelian ratio equal to 3. As far as the derivation of the DGL theory is based on the Abelian projected gluodynamics, this seems to be unavoidable in the DGL theory, too. However, in a lattice investigation for SU(2) Abelian projected gluodynamics, Poulis [12] has found the ratio between the string tensions of the adjoint and fundamental represen-

tations, somewhere between the Abelian and Casimir scaling. This result is encouraging for the Abelian projected models to be able to provide the Casimir scaling. The case of SU(2) gluodynamics has been considered in Ref. [13] in the context of an extended effective theory. It is discussed there that in the London limit Casimir scaling can be expected to hold. In this paper we examine straightforwardly the DGL theory for SU(3) gluodynamics with respect to the string tensions for various external charges without further modifications.

Considering the DGL theory just at a phenomenological level, it might be natural to restrict its application to mesonic [10,14,15], baryonic [15,16], glueball [17] and perhaps to exotic states, and it would seem inappropriate to apply it to the so-called gluelump bound states made of infinitely heavy adjoint charges. However, because of the current interest in this issue, it is interesting to discuss how this kind of string would be represented within the DGL theory, and then to answer the question whether the Casimir scaling poses really a problem or not.

In this paper we compute the string tensions of flux tubes which are originating from various dimensions of representation of color charge within the DGL theory. For this purpose we adopt the manifestly Weyl symmetric approach [15,18], which will turn out to be very useful for classifying the flux tube in various representations. Finally, based on these results as a function of the mass ratio between dual gauge bosons and monopoles, we would like to discuss how good the Casimir scaling can be accomplished within the DGL theory.

II. WEYL SYMMETRIC FORMULATION OF THE DGL THEORY

The DGL Lagrangian [10] is given by [1]

$$\begin{aligned}\mathcal{L}_{\rm DGL} = & -\frac{1}{4}\left((\partial\wedge\vec{B})_{\mu\nu}+e\vec{\Sigma}_{\mu\nu}\right)^2 \\ & +\sum_{i=1}^{3}\left[\left|\left(\partial_\mu+ig\vec{\epsilon}_i\cdot\vec{B}_\mu\right)\chi_i\right|^2-\lambda\left(|\chi_i|^2-v^2\right)^2\right],\end{aligned} \tag{2.1}$$

where $\vec{B}_\mu$ and χ_i denote the dual gauge field with two components (B_μ^3, B_μ^8) and the complex scalar monopole field, respectively. The quark current $\vec{j}_\mu=\bar{q}\gamma_\mu\vec{H}q$, where $\vec{H}=(T_3,T_8)$, is represented by the boundary of a nonlocal string term $\vec{\Sigma}_{\mu\nu}$, which expresses the color-electric Dirac string singularity through the modified dual Bianchi identity $\partial^{\nu\,*}\vec{\Sigma}_{\mu\nu}=$

[1] Throughout this paper, we use the following notations: Latin indices i, j express the labels 1,2,3, which are not to be summed over unless explicitly stated. Boldface letters, which appear later, denote three-vectors.

$\vec{j}_\mu$. Note that $(\partial \wedge \vec{B})_{\mu\nu} \equiv \partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu$ satisfies $\partial^{\nu *}(\partial \wedge \vec{B})_{\mu\nu} = 0$. Since the diagonal component of the matrix $\vec{H}$ gives the weight vector of the SU(3) algebra $\vec{w}_j$ $(j = 1, 2, 3)$, where $\vec{w}_1 = \left(1/2, \sqrt{3}/6\right), \vec{w}_2 = \left(-1/2, \sqrt{3}/6\right), \vec{w}_3 = \left(0, -1/\sqrt{3}\right)$, one can define the color-electric charges of the quarks as $\vec{Q}_j^{(e)} \equiv e\vec{w}_j$. Here, $j = 1, 2, 3$ correspond to the color-electric charges, red (R), blue (B), and green (G). Accordingly we can write the nonlocal term as $e\vec{\Sigma}_{\mu\nu} = e\vec{w}_j \Sigma^{(e)}_{j\,\mu\nu}$. On the other hand, the root vectors of the SU(3) algebra $\vec{\epsilon}_i$ are used to define the color-magnetic charges of the monopole field as $Q_i^{(m)} \equiv g\vec{\epsilon}_i$ $(i = 1, 2, 3)$, where $\vec{\epsilon}_1 = \left(-1/2, \sqrt{3}/2\right), \vec{\epsilon}_2 = \left(-1/2, -\sqrt{3}/2\right), \vec{\epsilon}_3 = (1, 0)$. Both color-electric and color-magnetic charges satisfy the extended Dirac quantization condition $\vec{Q}_i^{(m)} \cdot \vec{Q}_j^{(e)} = 2\pi m_{ij}$ $(eg = 4\pi)$. Here m_{ij} is an integer following the definition

$$m_{ij} = 2\vec{\epsilon}_i \cdot \vec{w}_j = \sum_{k=1}^{3} \epsilon_{ijk} = \{0, 1, -1\}, \tag{2.2}$$

where ϵ_{ijk} is the 3rd-rank antisymmetric tensor. Typical mass scales in the DGL theory are the mass of the dual gauge field $m_B = \sqrt{3}gv$ and of the monopole field $m_\chi = 2\sqrt{\lambda}v$. Their ratio, the so-called Ginzburg-Landau (GL) parameter $\kappa \equiv m_\chi/m_B$, characterizes the type of dual "superconductivity" of the vacuum. Like in real superconductive materials, the properties of the vacuum might be very different depending on the actual value of κ.

We make the Weyl symmetry of the DGL theory (2.1) manifest, with the help of an extended dual gauge field [15,18], defined by

$$B_{i\,\mu} \equiv g\vec{\epsilon}_i \cdot \vec{B}_\mu. \qquad (i = 1, 2, 3) \tag{2.3}$$

Here, a constraint $\sum_{i=1}^{3} B_{i\,\mu} = 0$ appears, since $\sum_{i=1}^{3} \vec{\epsilon}_i = 0$. The DGL Lagrangian (2.1) is now written as

$$\mathcal{L}_{\rm DGL} = \sum_{i=1}^{3} \left[-\frac{1}{4g_m^2} {}^*F_{i\,\mu\nu}{}^2 + \left|\left(\partial_\mu + iB_{i\,\mu}\right)\chi_i\right|^2 - \lambda\left(|\chi_i|^2 - v^2\right)^2 \right], \tag{2.4}$$

$${}^*F_{i\,\mu\nu} \equiv (\partial \wedge B_i)_{\mu\nu} + 2\pi \sum_{j=1}^{3} m_{ij} \Sigma^{(e)}_{j\,\mu\nu}, \tag{2.5}$$

where the dual gauge coupling g is scaled as

$$g_m \equiv \sqrt{\frac{3}{2}}g. \tag{2.6}$$

The factor 2π in front of the Dirac string term is derived from the Dirac quantization condition. Clearly, the expression (2.4) is manifestly Weyl symmetric since all indices i and j are summed over. Apparently the dual gauge symmetry is extended to $[\mathrm{U}(1)]^3$, achieved by a set of transformation

$$\chi_i \to \chi_i e^{if_i}, \quad \chi_i^* \to \chi_i^* e^{-if_i},$$
$$B^{\mathrm{reg}}_{i\,\mu} \to B^{\mathrm{reg}}_{i\,\mu} - \partial_\mu f_i, \qquad (i = 1, 2, 3). \tag{2.7}$$

However, the number of gauge degrees of freedom is not enlarged because of the constraint $\sum_{i=1}^{3} B_{i\,\mu} = 0$.

In what follows we investigate the flux-tube solutions related to a separated quark and antiquark pair and related to analogous states (with higher representation charges) within the DGL theory. In order to find such solutions it is useful to dispose the behavior of the dual gauge field, which can be achieved by the decomposition of the dual gauge field into two parts, the regular (no Dirac string) part and the singular (Dirac string) part [18],

$$B_{i\,\mu} \equiv B^{\mathrm{reg}}_{i\,\mu} + \sum_{j=1}^{3} m_{ij} B^{\mathrm{sing}}_{j\,\mu} \quad (i = 1, 2, 3), \tag{2.8}$$

where the singular part is determined so as to define the color-electric charge density $C^{(e)}_{j\,\mu\nu}$ as

$$(\partial \wedge B^{\mathrm{sing}}_j)_{\mu\nu} + 2\pi \Sigma^{(e)}_{j\,\mu\nu} = 2\pi C^{(e)}_{j\,\mu\nu}, \quad (j = 1, 2, 3). \tag{2.9}$$

The explicit form of $C^{(e)}_{j\,\mu\nu}$ is given by

$$C^{(e)}_{j\,\mu\nu}(x) = \frac{1}{4\pi^2} \int d^4y \frac{1}{|x-y|^2} {}^*(\partial \wedge j^{(e)}_j(y))_{\mu\nu}, \tag{2.10}$$

where $j^{(e)}_{j\,\mu} = \partial^\nu {}^*\Sigma^{(e)}_{j\,\mu\nu}$. Note that if there is no quark source, we do not need to have a singular part $B^{\mathrm{sing}}_{j\,\mu}$. Thus, the dual field strength tensor is rewritten as

$${}^*F_{i\,\mu\nu} = (\partial \wedge B^{\mathrm{reg}}_i)_{\mu\nu} + 2\pi \sum_{j=1}^{3} m_{ij} C^{(e)}_{j\,\mu\nu}. \tag{2.11}$$

In the static q-$\bar{q}$ system, $C^{(e)}_{j\,\mu\nu}$ turns out to be the Coulombic color-electric field originating from the color-electric charge.

III. THE STRING TENSION OF THE FLUX TUBE

Let us now consider an idealized system, an infinitely long flux tube with cylindrical and translational symmetry. In this case, the terms related to $C^{(e)}_{j\,\mu\nu}$ can be neglected since they are relevant only for short separation of quark and antiquark. One finds that the integration of square of $C^{(e)}_{j\,\mu\nu}$ gives the Coulomb energy including the self-energy of the color-electric charge. Correspondingly, now we only pay attention to the energy per length (string tension) of the flux tube which has the terminating charges at infinity. In

order to classify the types of the flux tube, we use a notation analogous to the q-$\bar{q}$ system. The fields depend only on the radial coordinate r as

$$\phi_i = \phi_i(r), \quad \boldsymbol{B}_i^{\rm reg} = B_i^{\rm reg}(r)\boldsymbol{e}_\varphi \equiv \frac{\tilde{B}_i^{\rm reg}(r)}{r}\boldsymbol{e}_\varphi, \tag{3.1}$$

where $\phi_i(r)$ is the modulus of the monopole field $\chi_i = \phi_i \exp(i\eta_i)$, and φ denotes the azimuthal angle. Note that the phase of the monopole field is now assumed to be regular $[\partial_\mu, \partial_\nu]\eta_i = 0$, which is absorbed into the regular part of the dual gauge field by the replacement $B_i^{\rm reg} + \partial_\mu \eta_i \to B_i^{\rm reg}$. If $[\partial_\mu, \partial_\nu]\eta_i \neq 0$, this produces the closed color-electric Dirac string singularity [17]. Putting the quark at $z = -\infty$ and antiquark at $z = \infty$ (this leads to the factor minus in $\boldsymbol{B}_i^{\rm sing}$), the solution of (2.9) is easily found to be

$$\boldsymbol{B}_i^{\rm sing} = -\frac{n_i^{(m)}}{r}\boldsymbol{e}_\varphi. \tag{3.2}$$

Here $n_i^{(m)}$ is an integer corresponding to the number of color-electric Dirac strings in the dual gauge field within the Weyl symmetric representation, which is expressed by the relation

$$n_i^{(m)} \equiv \sum_{j=1}^{3} m_{ij} n_j^{(e)}. \tag{3.3}$$

Here $n_j^{(e)}$ is the modulo 2π of $\Sigma_{j\,\mu\nu}^{(e)}$, the number of j-type color-electric charges attached to both ends of the flux tube. Various dimensions of the representation of charges in SU(3) group and corresponding winding numbers are classified in Table. I. For instance, the fundamental representation ($D = \mathbf{3}$) has three different charges R, B, and G. These charges have the numbers $(n_1^{(e)}, n_2^{(e)}, n_3^{(e)}) = (1,0,0)$, $(0,1,0)$, and $(0,0,1)$, which are also written by using the expression (3.3) as $(n_1^{(m)}, n_2^{(m)}, n_3^{(m)}) = (0,-1,1)$, $(1,0,-1)$, and $(-1,1,0)$, respectively. These rules hold similarly for the higher dimension of the representation. However one should take into account the relation $RB = \bar{G}$, $BG = \bar{R}$, $GR = \bar{B}$ and $RBG = 0$ following the definition of the fundamental color-electric charge with the weight vectors of the SU(3) algebra. Making use of these, some of the charges classified in the higher dimension of the representation are reduced into that of the lower ones.

Then, the field equations are the following

$$\frac{d^2 \tilde{B}_i^{\rm reg}}{dr^2} - \frac{1}{r}\frac{d\tilde{B}_i^{\rm reg}}{dr} - 2g_{\rm m}^2 \left(\tilde{B}_i^{\rm reg} - n_i^{(m)}\right)\phi_i^2 = 0, \tag{3.4}$$

$$\frac{d^2\phi_i}{dr^2} + \frac{1}{r}\frac{d\phi_i}{dr} - \left(\frac{\tilde{B}_i^{\rm reg} - n_i^{(m)}}{r}\right)^2 \phi_i - 2\lambda\phi_i(\phi_i^2 - v^2) = 0. \tag{3.5}$$

The string tension is the energy of the flux tube per unit length,

$$\sigma_D = 2\pi \sum_{i=1}^{3} \int_0^\infty r dr \left[\frac{1}{2g_\mathrm{m}^2} \left(\frac{1}{r} \frac{d\tilde{B}_i^\mathrm{reg}}{dr} \right)^2 + \left(\frac{d\phi_i}{dr} \right)^2 \right.$$
$$\left. + \left(\frac{\tilde{B}_i^\mathrm{reg} - n_i^{(m)}}{r} \right)^2 \phi_i^2 + \lambda (\phi_i^2 - v^2)^2 \right], \tag{3.6}$$

To make the energy of the system finite, we have to postulate the boundary conditions:

$$\tilde{B}_i^\mathrm{reg} = 0, \quad \phi_i = \begin{cases} 0 & \left(n_i^{(m)} \neq 0\right) \\ v & \left(n_i^{(m)} = 0\right) \end{cases} \qquad \text{as} \qquad r \to 0,$$
$$\tilde{B}_i^\mathrm{reg} = n_i^{(m)}, \quad \phi_i = v \qquad \text{as} \qquad r \to \infty. \tag{3.7}$$

For an analytical evaluation of the string tension, it is useful to rewrite the expression (3.6) in the form [18],

$$\sigma_D = 2\pi v^2 \sum_{i=1}^{3} \left| n_i^{(m)} \right| + 2\pi \sum_{i=1}^{3} \int_0^\infty r dr \left[\frac{1}{2g_\mathrm{m}^2} \left(\frac{1}{r} \frac{d\tilde{B}_i^\mathrm{reg}}{dr} \pm g_\mathrm{m}^2 (\phi_i^2 - v^2) \right)^2 \right.$$
$$\left. + \left(\frac{d\phi_i}{dr} \pm \left(\tilde{B}_i^\mathrm{reg} - n_i^{(m)} \right) \frac{\phi_i}{r} \right)^2 + \frac{1}{2} \left(2\lambda - g_\mathrm{m}^2 \right) (\phi_i^2 - v^2)^2 \right]. \tag{3.8}$$

From this expression we find that in the Bogomol'nyi limit [18,19],

$$g_\mathrm{m}^2 = 2\lambda, \quad \text{or} \quad 3g^2 = 4\lambda, \tag{3.9}$$

which corresponds to $\kappa = m_\chi / m_B = 1$, a considerable simplification occurs. The vacuum is separated into two types at this Bogomol'nyi point, type-I ($\kappa < 1$) and type-II ($\kappa > 1$) in analogy to the superconducting material. In the Bogomol'nyi limit one gets the saturated string tension,

$$\sigma_D = 2\pi v^2 \sum_{i=1}^{3} \left| n_i^{(m)} \right|. \tag{3.10}$$

One finds that the string tension for the fundamental representation becomes $\sigma_3 = 4\pi v^2$ since $\sum_{i=1}^{3} \left| n_i^{(m)} \right| = 2$. In this special case, the profiles of the dual gauge field and the monopole field are determined by the first-order differential equations,

$$\frac{1}{r} \frac{d\tilde{B}_i^\mathrm{reg}}{dr} \pm g_\mathrm{m}^2 (\phi_i^2 - v^2) = 0, \tag{3.11}$$
$$\frac{d\phi_i}{dr} \pm \left(\tilde{B}_i^\mathrm{reg} - n_i^{(m)} \right) \frac{\phi_i}{r} = 0. \tag{3.12}$$

These field equations reproduce the second-order differential equations (3.4) and (3.5) when the relation (3.9) is fulfilled. Note that the procedure to find the Bogomol'nyi limit is an extension of the method used for the U(1) Abelian Higgs model [20,21] to the U(1)×U(1) DGL theory corresponding to SU(3) gluodynamics in Abelian projection.

Finally, let us compute the ratio of the string tension between the higher and fundamental representations. In the Bogomol'nyi limit, this can be done easily by using the expression (3.10). For the ratio between $D = \mathbf{8}$ and $D = \mathbf{3}$, we get $d_8 \equiv \sigma_8/\sigma_3 = (2\pi v^2 \times 4)/(2\pi v^2 \times 2) = 2$. In general, one can recognize a simple rule

$$d_D \equiv \frac{\sigma_D}{\sigma_3} = p + q. \tag{3.13}$$

Here $p + q$ is the sum of weight factors in the SU(3) representation, which physically corresponds to the number of the color-electric Dirac string inside the flux tube in the framework of the DGL theory. In the type-I ($\kappa < 1$) or type-II ($\kappa > 1$) parameter range, we have to calculate the expression (3.6) by solving the field equations (3.4) and (3.5) numerically. The corresponding numerical results are shown in Figs. 1−3. In Fig. 1 we show the values of d_8 and d_6 corresponding to $p+q = 2$ as a function of the GL parameter. Similarly, in Figs. 2 and 3, the ratios d_D classified in the representation $p + q = 3$ and $p + q = 4$ are shown, respectively. The dotted line denotes the ratio of the quadratic Casimir charges: $C^{(2)}(D)/C^{(2)}(F = \mathbf{3}) = 2.25$, 2.5, 4, 4.5, 6, 6.25, 7 for the dimensions $D =$ **8**, **6**, **15a**, **10**, **27**, **24**, **15s**, respectively. Of course, at $\kappa = 1$ the analytical results (3.13) are realized, and all ratios increase monotonously with the GL parameter κ. It is interesting to note that the Casimir scaling is accomplished only in the type-II region, for $\kappa = 5 \sim 9$, depending on the Casimir ratio for a particular representation. It seems that the DGL theory in its present shape can not describe the Casimir scaling for all representations simultaneously, with a unique value of κ. Some additional terms will be necessary to slightly modify the influence of κ on the energy of the flux tube solutions corresponding to various external charges.

IV. SUMMARY AND DISCUSSION

We have studied the string tension of flux tubes associated with static charges in higher SU(3) representations within the dual Ginzburg-Landau theory in a manifestly Weyl symmetric approach. We have found that the ratio of the string tension between higher and fundamental representations, $d_D \equiv \sigma_D/\sigma_F$, depends only on the ratio between the monopole mass m_χ and the mass of the dual gauge boson m_B, the Ginzburg-Landau parameter $\kappa = m_\chi/m_B$. We have pointed out that the ratios d_D have a simple form in the case of the Bogomol'nyi limit in terms of the number of color-electric Dirac strings inside the flux tube. We have numerically determined the ratios in the type-I ($\kappa < 1$) and type-II ($\kappa > 1$) parameter ranges and found them monotonically rising with κ. In principle, such deviation of the ratio from the number of color-electric Dirac string can be understood as the effect of the flux-tube interaction. In the type-II parameter range, in the interval $\kappa = 5 \sim 9$, the Casimir scaling is approximately reproduced for all representations D studied in the present paper. This shows that it was premature to say that the the

dual superconducting scenario of confinement is in plain contradiction with the Casimir scaling.

ACKNOWLEDGMENT

The authors acknowledge fruitful discussions with M. Takayama. E.-M.I. is grateful for the support by the Ministry of Education, Culture and Science of Japan (Monbu-Kagaku-sho). T.S. acknowledges the financial support from JSPS Grant-in Aid for Scientific Research (B) No. 10440073 and No. 11695029.

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TABLES

TABLE I. Classification of the color-electric charges and the winding numbers of flux tubes for various dimensions of the representations of SU(3) group. D denotes the dimension of representation, (p, q) the weight factors. Q is the color-electric charge of Q-$\bar{Q}$ flux-tube system, where the number of the color-electric Dirac strings attached to the charge Q is $n_j^{(e)}$. The number of strings in the dual gauge field within the Weyl symmetric representation is given by $n_i^{(m)} = \sum_{j=1}^{3} m_{ij} n_j^{(e)}$, where $m_{ij} = 2\vec{\epsilon}_i \cdot \vec{w}_j$.

D	(p,q)	$p+q$	Q	$n_1^{(e)}$	$n_2^{(e)}$	$n_3^{(e)}$	$n_1^{(m)}$	$n_2^{(m)}$	$n_3^{(m)}$
3	(1,0)	1	R	1	0	0	0	−1	1
			B	0	1	0	1	0	−1
			G	0	0	1	−1	1	0
8	(1,1)	2	$R\bar{B}$	1	−1	0	−1	−1	2
			$B\bar{G}$	0	1	−1	2	−1	−1
			$G\bar{R}$	−1	0	1	−1	2	−1
6	(2,0)	2	RR	2	0	0	0	−2	2
			BB	0	2	0	2	0	−2
			GG	0	0	2	−2	2	0
15a	(2,1)	3	$RR\bar{B}$	2	−1	0	−1	−2	3
			$BB\bar{G}$	0	2	−1	3	−1	−2
			$GG\bar{R}$	−1	0	2	−2	3	−1
			$RR\bar{G}$	2	0	−1	1	−3	2
			$BB\bar{R}$	−1	2	0	2	1	−3
			$GG\bar{B}$	0	−1	2	−3	2	1
10	(3,0)	3	RRR	3	0	0	0	−3	3
			BBB	0	3	0	3	0	−3
			GGG	0	0	3	−3	3	0
27	(2,2)	4	$RR\bar{B}\bar{B}$	2	−2	0	−2	−2	4
			$BB\bar{G}\bar{G}$	0	2	−2	4	−2	−2
			$GG\bar{R}\bar{R}$	−2	0	2	−2	4	−2
24	(3,1)	4	$RRR\bar{B}$	3	−1	0	−1	−3	4
			$BBB\bar{G}$	0	3	−1	4	−1	−3
			$GGG\bar{R}$	−1	0	3	−3	4	−1
			$RRR\bar{G}$	3	0	−1	1	−4	3
			$BBB\bar{R}$	−1	3	0	3	1	−4
			$GGG\bar{B}$	0	−1	3	−4	3	1
15s	(4,0)	4	$RRRR$	4	0	0	0	−4	4
			$BBBB$	0	4	0	4	0	−4
			$GGGG$	0	0	4	−4	4	0

FIGURES

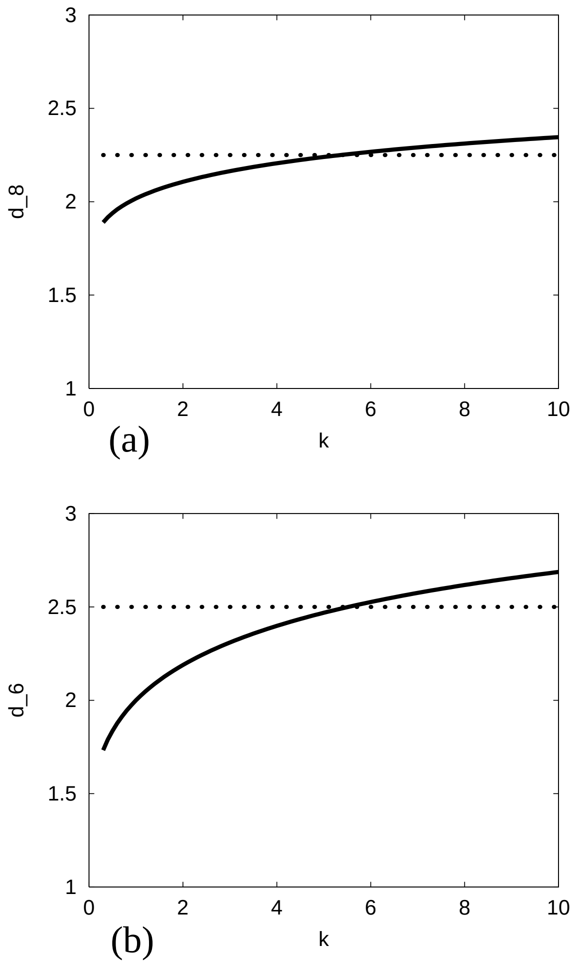

FIG. 1. The ratio of the string tension (a) for the octet representation $d_8 = \sigma_8/\sigma_3$ (the dotted line marks the ratio of quadratic Casimir charges $C^{(2)}(\mathbf{8})/C^{(2)}(\mathbf{3}) = 2.25$) and (b) for the sextet representation $d_6 = \sigma_6/\sigma_3$ ($C^{(2)}(\mathbf{6})/C^{(2)}(\mathbf{3}) = 2.5$). The weight factor is $p + q = 2$. Casimir scaling for the values of d_8 and d_6 is observed at $\kappa \approx 5$.

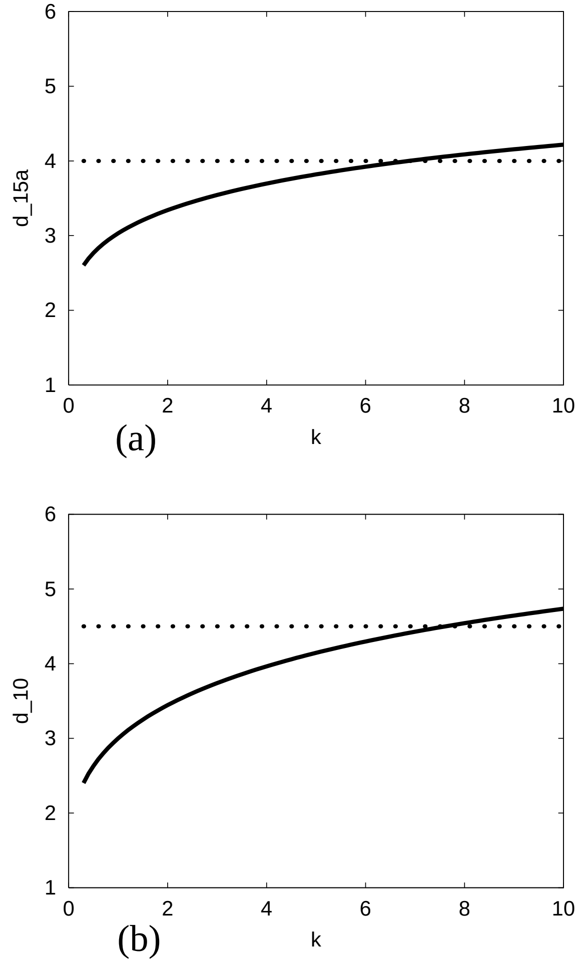

FIG. 2. Similar ratios as in Fig. 1, (a) $d_{15a} = \sigma_{15a}/\sigma_3$ $(C^{(2)}(\mathbf{15a})/C^{(2)}(\mathbf{3}) = 4)$ and (b) $d_{10} = \sigma_{10}/\sigma_3$ $(C^{(2)}(\mathbf{10})/C^{(2)}(\mathbf{3}) = 4.5)$. The weight factor is $p + q = 3$. Casimir scaling for the values of d_{15a} and d_{10} is observed at $\kappa \approx 7$.

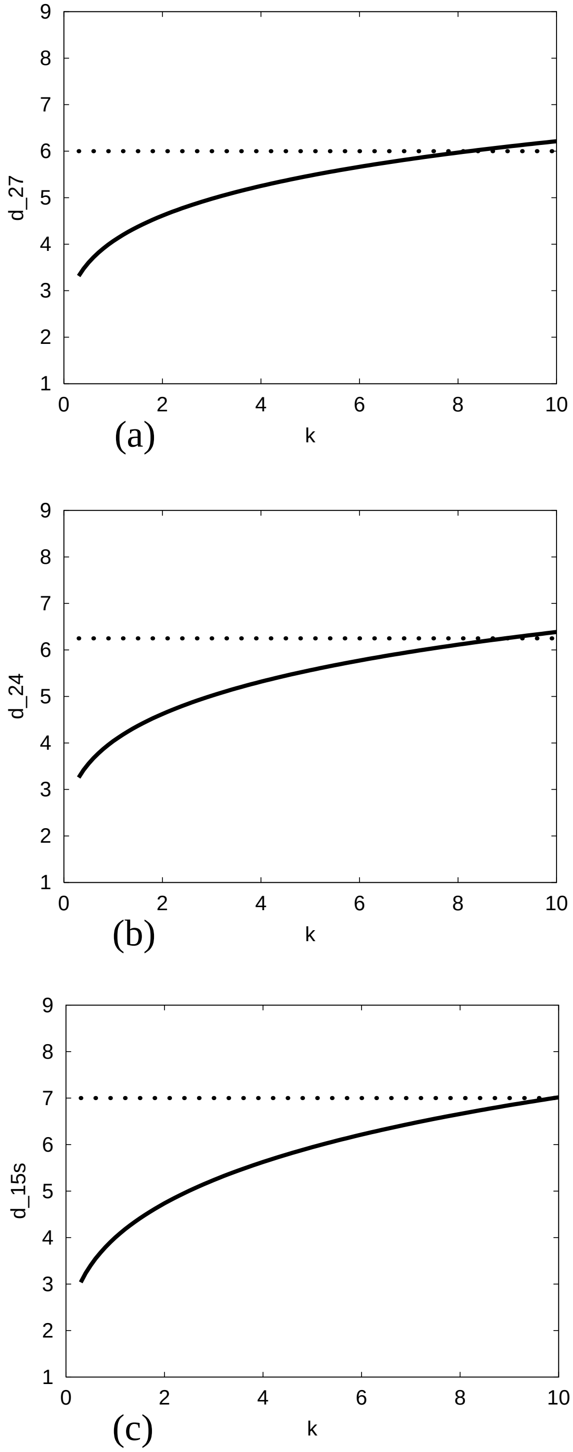

FIG. 3. Similar ratios as in Fig. 1, (a) $d_{27} = \sigma_{27}/\sigma_3$ ($C^{(2)}(\mathbf{27})/C^{(2)}(\mathbf{3}) = 6$), (b) $d_{24} = \sigma_{24}/\sigma_3$ ($C^{(2)}(\mathbf{24})/C^{(2)}(\mathbf{3}) = 6.25$), and (c) $d_{15s} = \sigma_{15s}/\sigma_3$ ($C^{(2)}(\mathbf{15s})/C^{(2)}(\mathbf{3}) = 7$). The weight factor is $p + q = 4$. Casimir scaling for the values of d_{27}, d_{24}, and d_{15s} is observed at $\kappa \approx 9$.